The placement exam will consist of up to 30 problems, each of which is similar to one of the following problems. If you can do problems like these listed below, you should have no problem with the pretest. You may not use a calculator on the exam, and you will have up to 60 minutes to complete the exam.

Questions

Values of trigonometric and inverse trigonometric functions

1. Find \( \cos \left( -\frac{\pi}{6} \right) \).
2. Find \( \csc \left( \frac{5\pi}{6} \right) \).
3. Find \( \sin \left( \frac{3\pi}{4} \right) \).
4. Find \( \tan \left( \frac{11\pi}{6} \right) \).
5. Find \( \sin 0 \).
6. Find \( \sec \left( \frac{7\pi}{6} \right) \).
7. Find \( \cot \left( \frac{4\pi}{3} \right) \).
8. Find \( \cos \pi \).
9. Find \( \tan \left( \frac{3\pi}{4} \right) \).
10. Find \( \sin^{-1} 1 \)
11. Find \( \cos^{-1} \left( -\frac{1}{\sqrt{2}} \right) \).
12. Find \( \tan^{-1} -\sqrt{3} \).
13. Find the exact value of \( \cos(\arctan \frac{1}{3}) \)
14. Simplify \( \sin(\cos^{-1}(x)) \)

Trig identities

15. Give the formula for \( \cos^2 x \) in terms of \( \sin^2 x \).
16. Give the formula for \( \cos(2x) \) in terms of \( \cos x \) and \( \sin x \).
17. Give the formula for \( \sec^2 x \) in terms of \( \tan^2 x \).
18. Give the formula for \( \sin(2x) \) in terms of \( \cos x \) and \( \sin x \).

Applications of trig functions

19. You have a triangle with angles \( \alpha = \frac{\pi}{4} \) and \( \beta = \frac{\pi}{2} \). The side opposite \( \beta \) has length 7 inches. How long is the side opposite \( \alpha \)?
20. You have a triangle with angles \( \alpha = \frac{\pi}{3} \) and \( \beta = \frac{\pi}{2} \). The side opposite \( \beta \) has length 13 inches. How long is the side opposite \( \alpha \)?

**Inverses**

21. Let \( f(x) = \frac{3x-7}{11x+5} \). Find \( f^{-1}(x) \).

22. Suppose \( rx^2 + sx + t = 0 \) where \( r \neq 0 \). Then \( x = \)

23. Find all solutions to \( x^4 - 11x^2 + 30 = 0 \).

24. Find all solutions to \( x^2 + 5x + 6 = 0 \).

25. Suppose \( -2x + y = -4 \) and \( x - 4y = -4 \). Find \( x - y \).

26. For what value of \( k \) does the system of equations, \( 2x + y = 1 \), \( 12x + ky = 12 \) have no solutions?

**Exponential and Logarithmic Functions**

27. Expand the logarithmic expression: \( \ln \left( \frac{(x - 3)^3}{(x - 3)(2x - 9)^6} \right) \).

28. Expand the logarithmic expression: \( \ln \sqrt{\frac{x^2 - 1}{x^2 + 1}} \).

29. Solve the equation: \( e^{3x/4} = 10 \).

30. Solve the equation: \( 7 - (5)4^{3x} = -18 \).

31. Solve the equation: \( \ln(4x - 17) = 15 \).

**Word problems**

32. A water tank is initially \( \frac{1}{4} \) full. After adding 14 gallons of water, it is \( \frac{7}{8} \) full. What is the capacity of the tank in gallons?

33. The area of a triangle is 3 more than the height of the triangle. If the base of the triangle is 15 inches, what is the height?

34. A man 6 feet tall stands 12 feet away from a lamppost. If his shadow is 4 feet long, how tall is the lamppost?

35. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. Write an expression in one variable that gives the area of the field.

36. A cylindrical can is to be made to hold 1 L or 1000 cm\(^3\) of oil. Find an expression in terms of the radius of the can that gives the surface area of the can.

**Solving inequalities**

37. Solve \( x(x + 5)(x - 4) < 0 \) for \( x \).

38. Solve \( |5x - 25| < 20 \) for \( x \).

39. Find the solution to the inequality, \( |10x - 2| < 5 \).

**Solving equations**

40. Let \( y = \frac{5x+5}{3x^2+7} \). Solve for \( x \).

41. Solve for \( x \) in the equation \( 10(1 + 17x)^{1/2} + 85x(1 + 17x)^{-1/2} = 0 \).

42. Find all positive solutions to \( \sqrt{x^2 + 9} = 8 \).

**Simplifying expressions**

43. Simplify \( \frac{2}{x+1} - \frac{3}{x-3} \).

44. Simplify \( (57)^{-1/7} \).

45. Simplify \( \frac{2xy^3}{x^2+y^4} \).

46. Simplify \( (8)^{2/3}(16)^{1/4} \).
47. Simplify \( \frac{1}{x^2+4} + \frac{2}{x} \).

48. Simplify \( \frac{1}{x^4-1} + \frac{2}{3x-6} \).

49. Simplify \( \frac{3x-4}{x-y} \).

50. Simplify \( |\sqrt{7} - 7| - | - 5| \).

51. Simplify \( |\frac{1}{8} - \frac{1}{7}\sqrt{2}| - | - 4| \).

52. Simplify \( \frac{x^2-4}{3x} \cdot \frac{30}{3x-6} \).

53. Suppose \( f(x) = x^2 + 6x \). Simplify \( \frac{f(x+h)-f(x)}{h} \).

**Symmetry**

54. Identify which functions below have even symmetry, odd symmetry, or neither.

(a) \( f(x) = x^2 + 9 \)

(b) \( f(x) = x^2 + 3x + 4 \)

(c) \( f(x) = \cos x \)

(d) \( f(x) = \tan x \)

(e) \( f(x) = 8x^5 - 3x + 7 \)

(f) \( f(x) = \sin x \)

(g) \( f(x) = \frac{3x^2 + 11}{(x-1)^2} \)

**Factoring of polynomials**

55. Factor the polynomial \( x^2 + 13x + 42 \).

56. Factor the polynomial \( 5x^2 - 15x - 20 \).

57. Factor the polynomial \( 3y(x-3) - 2(x-3) \).

58. Factor the polynomial \( 6x^2 + 43x + 42 \).

59. Factor the polynomial \( x^2 - 9 \).

**Composition of Functions**

60. Suppose \( f(-2) = 3, f(3) = -2, \) and \( f(1) = -1, \) while \( g(3) = 1, g(-2) = 2, \) and \( g(1) = -2 \). Find \( f(g(-2 + 3)) \).

61. Let \( f(x) = 4x^2 - 7x \) and \( g(x) = -5x^2 \). Find \( f(g(1)) \).

**Lines**

62. A line having the equation \( y = mx + b \) passes through the points \((-4, -7) \) and \((9,1) \). Find \( m + b \).

63. Find the equation of the line through the points \((-4,3) \) and \((5,4) \).

64. Find the equation of the line perpendicular to \(-12x + 7y = 11 \) through \((4,2) \).

65. Find the equation of the line parallel to \(-3x + y = 3 \) and passing through \((0,8) \).

66. A line has the equation \(6x = 5y + 10 \). What is its slope?

**Miscellaneous problems**

67. Find the distance between the two points \((2, -3) \) and \((-5, 0) \).

68. Which number is bigger, \( \frac{1}{3} \) or \( \frac{4}{16} \)?

69. What is the range of the function \( f(x) = 10(x + 7)^2 + 6 \)?

70. If \( z = (4x + 5y)^2 \), then \( z = \)

71. Find the domain of the function \( f(x) = \sqrt{\frac{2x+5}{3x-2}} \).
72. Simplify as much as possible $\sqrt{x^2 + y^2}$.

**Law of cosines and sines**

73. In the picture, $\theta = \frac{\pi}{4}$, $\alpha = \frac{2\pi}{3}$, and $c = 2$. Find $a$.

74. In the picture, $a = 5$, $b = 3$, and $\theta = \frac{\pi}{6}$. Find $c$.

**Answers**

1. $\frac{\sqrt{3}}{2}$
2. $2$
3. $\frac{\sqrt{3}}{2}$
4. $\frac{1}{\sqrt{3}}$
5. $0$
6. $-\frac{2}{\sqrt{3}}$
7. $\frac{1}{\sqrt{3}}$
8. $-1$
9. $-1$
10. $\frac{\pi}{2}$
11. $\frac{2\pi}{3}$
12. $-\frac{\pi}{3}$
13. $\frac{3}{5}$
14. $\sqrt{1 - x^2}$
15. \( \cos^2 x = 1 - \sin^2 x \)
16. \( \cos(2x) = \cos^2 x - \sin^2 x \)
17. \( \sec^2 x = \tan^2 x + 1 \)
18. \( \sin(2x) = 2 \cos x \sin y \)
19. \( \frac{7}{\sqrt{2}} \)
20. \( \frac{13\sqrt{3}}{2} \)
21. \( f^{-1}(x) = \frac{5x + 7}{3 - 11x} \)
22. \( x = \frac{-x \pm \sqrt{x^2 - 4r^2}}{2r} \)
23. \( x = \pm \sqrt{6}, \pm \sqrt{5} \)
24. \( x = -2, -3 \)
25. \( \frac{8}{7} \)
26. 6
27. \( 3 \ln(x - 5) - \ln(x - 3) - 8 \ln(2x - 9) \)
28. \( \frac{1}{2} (\ln(x^2 - 1) - \ln(x^2 + 1)) \)
29. \( \frac{4}{3} \ln 10 \)
30. \( x = \frac{1}{3} \ln \frac{5}{4} \) or \( \frac{1}{3} \log_4 5 \)
31. \( x = \frac{1}{4} e^{15} + 3 \)
32. \( \frac{112}{5} \)
33. \( \frac{6}{13} \)
34. 30 feet
35. \( A = 1200x - \frac{1}{2} x^2 \), where \( x \) is the side length parallel to the river.
36. \( SA = 2000r^{-1} + 2\pi r^2 \)
37. \( (-\infty, -5) \cup (0, 4) \)
38. \((1, 9)\)
39. \( \frac{-3}{10} < x < \frac{7}{10} \)
40. \( x = \frac{-7y - 5}{7 - 3y} \)
41. \( x = -\frac{2}{11} \)
42. \( \sqrt{55} \)
43. \( -\frac{x + 9}{(x - 3)(x + 1)} \)
44. \( \frac{1}{5} \)
45. \( 8y \)
46. 8
47. \( \frac{2x^2 + x + 8}{(x^2 + 4)x} \)
48. \( \frac{2x + 7}{3(x - 2)(x + 2)} \)
49. \( \frac{-1}{xy} \)
50. \(-\sqrt{7} + 2\)
51. \(\frac{\sqrt{7}}{4} - \frac{33}{8}\)
52. \(\frac{2(x+2)}{x}\)
53. \(2x + 6 + h\)
54. (a) Even  
    (b) Neither  
    (c) Even  
    (d) Odd  
    (e) Neither  
    (f) Odd  
    (g) Neither
55. \((x + 7)(x + 6)\)
56. \(5(x - 4)(x + 1)\)
57. \((3y - 2)(x - 3)\)
58. \((6x + 7)(x + 6)\)
59. \((x - 3)(x + 3)\)
60. 3
61. 135
62. \(-\frac{51}{13}\)
63. \(y = \frac{1}{5}x + \frac{31}{5}\)
64. \(y = -\frac{7}{12}x + \frac{13}{3}\)
65. \(y = 3x + 8\)
66. \(\frac{6}{5}\)
67. \(\sqrt{58}\)
68. \(\frac{1}{3}\)
69. All real numbers \(\geq 6\)
70. \(16x^2 + 40xy + 25y^2\)
71. \((-\infty, -\frac{5}{2}) \cup (\frac{2}{5}, \infty)\)
72. The problem can not be simplified any more than it is.
73. \(\sqrt{6}\)
74. \(\sqrt{34 - 15\sqrt{3}}\)